Gaussian Elimination

Solve:

$$\begin{cases} x+y+2z=9 \quad (1) \\ 2x+4y-3z=1 \quad (2) \\ 3x+6y-5z=0 \quad (3) \end{cases}$$

Elim. x

$$\begin{cases} -2Eq(1)-2x-2y-4z=-18 \\ Eq.(2) \to 2x+4y-3z=1 \end{cases} \to ADD \to 2y-7z=-17 \quad (4)$$

$$\begin{cases} -3Eq(1) \to -3x-3y-6z=-27 \\ Eq.(3) \to \quad 3x+6y-5z=0 \end{cases} \to ADD \to 3y-11z=-27 \quad (5)$$

Elim. Y:

$$\begin{cases} \frac{1}{2}Eq(4) \to y-\frac{7}{2}z=-\frac{17}{2} \\ Eq.(5) \to \quad 3y-11z=-27 \end{cases} \quad \begin{cases} -3Eq(4) \to -3y+\frac{21}{2}z=\frac{51}{2} \\ Eq.(5) \to \quad 3y-11z=-27 \end{cases} \quad ADD \to -\frac{1}{2}z=-\frac{3}{2} \quad so \quad z=3$$

Augmented matrix

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{bmatrix}$$

Row operations

$$\cdots \begin{cases} -2R_1 + R_2 \to R_2 \\ -3R_1 + R_3 \to R_3 \end{cases} \Rightarrow \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 3 & -11 & -27 \end{bmatrix} \cdots \left\{ \frac{1}{2}R_2 \to R_2 \Rightarrow \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & -\frac{7}{2} & -\frac{17}{2} \\ 0 & 3 & -11 & -27 \end{bmatrix} \ldots \left\{ -3R_2 + R_3 \to R_3 \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & -\frac{7}{2} & -\frac{17}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} \end{bmatrix} \ldots \left\{ -2R_3 \to R_3 \right. \right. \right.$$

Row Echelon Form

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & -\frac{7}{2} & -\frac{17}{2} \\ 0 & 0 & 1 & 3 \end{bmatrix}$$